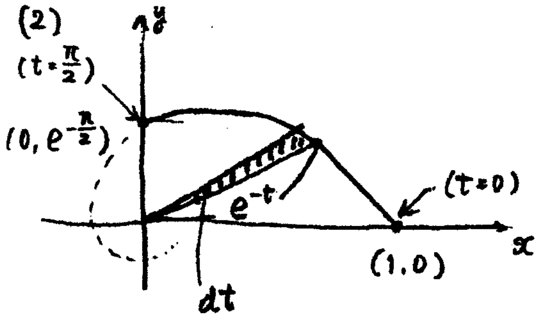
$$\begin{cases} x = e^{-t}\cos t \\ y = e^{-t}\sin t \end{cases} \quad \left(0 \leqq t \leqq \frac{\pi}{2}\right)$$

(1) $$\frac{dx}{dt} = -e^{-t}\cos t - e^{-t}\sin t$$
$$= -e^{-t}(\sin t + \cos t)$$

また $$\frac{dy}{dt} = -e^{-t}\sin t + e^{-t}\cos t$$
$$= -e^{-t}(\sin t - \cos t)$$ より

$$L = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$
$$= \int_0^{\frac{\pi}{2}} \sqrt{e^{-2t}\left\{(\sin t + \cos t)^2 + (\sin t - \cos t)^2\right\}}\, dt$$
$$= \int_0^{\frac{\pi}{2}} \sqrt{2e^{-2t}}\, dt$$
$$= \sqrt{2}\int_0^{\frac{\pi}{2}} e^{-t}\, dt$$
$$= \sqrt{2}\left[-e^{-t}\right]_0^{\frac{\pi}{2}} = \boxed{\sqrt{2}\left(-e^{-\frac{\pi}{2}} + 1\right)}$$

(2)


(2) 左図の微小三角形の面積は
$$\frac{1}{2}(e^{-t})^2 dt$$ より

求める面積は
$$\int_0^{\frac{\pi}{2}} \frac{1}{2}(e^{-t})^2 dt$$
$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}e^{-2t}\, dt$$
$$= \left[-\frac{1}{2}\times\frac{1}{2}e^{-2t}\right]_0^{\frac{\pi}{2}}$$
$$= \boxed{\frac{1}{4}\left(-e^{-\pi} + 1\right)}$$