

$$(1) \quad x = t + \sin t \quad \therefore \quad \frac{dx}{dt} = 1 + \cos t,$$

$$y = 1 - \cos t \quad \therefore \quad \frac{dy}{dt} = \sin t.$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{1 + \cos t}$$

(2) 求める長さを l とすると

$$l = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

$$= \int_0^{\pi} \sqrt{(1 + \cos t)^2 + \sin^2 t} dt$$

$$= \int_0^{\pi} \sqrt{1 + 2\cos t + \cos^2 t + \sin^2 t} dt$$

$$= \int_0^{\pi} \sqrt{2(1 + \cos t)} dt$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$= \int_0^{\pi} \sqrt{4 \cos^2 \left(\frac{t}{2}\right)} dt.$$

$\leftarrow (\because 0 \leq \frac{t}{2} \leq \frac{\pi}{2} \quad \therefore \quad \cos \frac{t}{2} > 0.)$

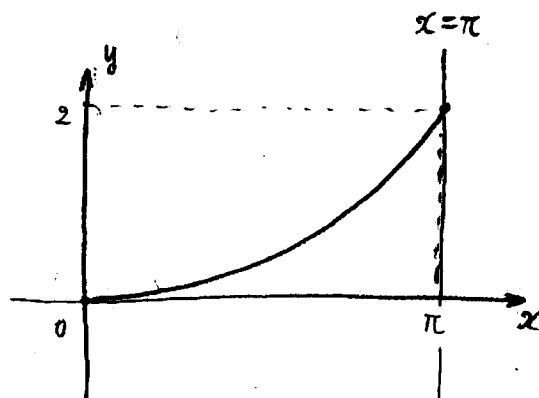
$$= \int_0^{\pi} 2 \cos \left(\frac{t}{2}\right) dt$$

$$= \left[4 \sin \frac{t}{2} \right]_0^{\pi}$$

$$= \boxed{4}$$

(3) 増減表をかくと

t	0	...	π
$\frac{dx}{dt}$	+	+	0
$\frac{dy}{dt}$	0	+	0
(x, y)	(0, 0)	$\nearrow$	$(\pi, 2)$



よって 求める面積を S とすると

$$\begin{aligned} S &= \int_0^{\pi} y \cdot dx \\ &= \int_0^{\pi} (1 - \cos t) \cdot (1 + \cos t) dt \\ &= \int_0^{\pi} (1 - \cos^2 t) dt \\ &= \int_0^{\pi} \sin^2 t dt \\ &= \int_0^{\pi} \frac{1 - \cos 2t}{2} dt \\ &= \left[\frac{t}{2} - \frac{1}{4} \sin 2t \right]_0^{\pi} \\ &= \boxed{\frac{\pi}{2}} \end{aligned}$$