

$$(1) \quad \sin \theta + \cos \theta = \frac{1}{2} \text{ の両辺を 2 乗して}$$

$$(\sin \theta + \cos \theta)^2 = \frac{1}{4} \text{ より}$$

$$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = \frac{1}{4}$$

$$1 + 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$2 \sin \theta \cos \theta = -\frac{3}{4}$$

$$\therefore \boxed{\sin \theta \cos \theta = -\frac{3}{8}}$$

$$(2) \quad \sin^3 \theta + \cos^3 \theta = (\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta)$$

$$= \frac{1}{2} \times \left(1 + \frac{3}{8}\right)$$

$$= \frac{1}{2} \times \frac{11}{8} = \boxed{\frac{11}{16}}$$

$$(3) \quad \sin^4 \theta + \cos^4 \theta = (\sin \theta + \cos \theta)(\sin^3 \theta + \cos^3 \theta) - \sin \theta \cos^3 \theta - \sin^3 \theta \cos \theta$$

$$= (\sin \theta + \cos \theta)(\sin^3 \theta + \cos^3 \theta) - \sin \theta \cos \theta (\sin^2 \theta + \cos^2 \theta)$$

$$= \frac{1}{2} \times \frac{11}{16} - \left(-\frac{3}{8}\right) \times 1$$

$$= \frac{11}{32} + \frac{12}{32} = \boxed{\frac{23}{32}}$$

$$(4) \quad \sin^5 \theta + \cos^5 \theta = (\sin^2 \theta + \cos^2 \theta)(\sin^3 \theta + \cos^3 \theta) - \sin^2 \theta \cos^3 \theta - \sin^3 \theta \cos^2 \theta$$

$$= 1 \times \frac{11}{16} - \sin^2 \theta \cos^2 \theta (\sin \theta + \cos \theta)$$

$$= \frac{11}{16} - \left(-\frac{3}{8}\right)^2 \times \frac{1}{2}$$

$$= \frac{11}{16} - \frac{9}{128}$$

$$= \frac{88-9}{128} = \boxed{\frac{79}{128}}$$