(1) $2\overrightarrow{V_{n+1}} = \overrightarrow{V_n} + \vec{c}$ より

$\overrightarrow{V_{n+1}} = \frac{1}{2}\overrightarrow{V_n} + \frac{1}{2}\vec{c}$ 　　　$\left(x = \frac{1}{2}x + \frac{1}{2}c \text{ より } x = c\right)$

両辺から $\vec{c}$ をひくと

$\overrightarrow{V_{n+1}} - \vec{c} = \frac{1}{2}(\overrightarrow{V_n} - \vec{c})$

ゆえに $\overrightarrow{V_n} - \vec{c} = (\overrightarrow{V_1} - \vec{c}) \times \left(\frac{1}{2}\right)^{n-1}$ より

$$\boxed{\overrightarrow{V_n} = \left(\frac{1}{2}\right)^{n-1}(\overrightarrow{V_1} - \vec{c}) + \vec{c}}$$

(2)(ア) $\overrightarrow{V_1} = (\cos 105^\circ, \sin 105^\circ)$, $\vec{c} = (\cos 15^\circ, -\sin 15^\circ) = (\cos(-15^\circ), \sin(-15^\circ))$ より

$$\begin{aligned}
\overrightarrow{V_n} \cdot \vec{c} &= \left\{\left(\frac{1}{2}\right)^{n-1}(\overrightarrow{V_1} - \vec{c}) + \vec{c}\right\} \cdot \vec{c} \\
&= \left(\frac{1}{2}\right)^{n-1}\overrightarrow{V_1} \cdot \vec{c} + \left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\}|\vec{c}|^2 \\
&= \left(\frac{1}{2}\right)^{n-1}\left\{\cos 105^\circ \cos(-15^\circ) + \sin 105^\circ \sin(-15^\circ)\right\} + \left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\} \times 1 \\
&= \left(\frac{1}{2}\right)^{n-1}\cos(105^\circ + 15^\circ) + 1 - \left(\frac{1}{2}\right)^{n-1} \\
&= \left(\frac{1}{2}\right)^{n-1} \times \left(-\frac{1}{2}\right) + 1 - \left(\frac{1}{2}\right)^{n-1} \\
&= -\frac{3}{2}\left(\frac{1}{2}\right)^{n-1} + 1 \\
&= \boxed{-3\left(\frac{1}{2}\right)^{n} + 1}
\end{aligned}$$

(イ)
$$\begin{aligned}
|\overrightarrow{V_n}|^2 &= \left|\left\{\left(\frac{1}{2}\right)^{n-1}(\overrightarrow{V_1} - \vec{c}) + \vec{c}\right\}\right|^2 \\
&= \left|\left(\frac{1}{2}\right)^{n-1}\overrightarrow{V_1} + \left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\}\vec{c}\right|^2 \\
&= \left(\frac{1}{2}\right)^{2(n-1)}|\overrightarrow{V_1}|^2 + 2\left(\frac{1}{2}\right)^{n-1}\left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\}\overrightarrow{V_1} \cdot \vec{c} + \left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\}^2|\vec{c}|^2 \\
&= \left(\frac{1}{2}\right)^{2(n-1)} + 2\left(\frac{1}{2}\right)^{n-1}\left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\}\left(1 - \frac{3}{2}\right) + 1 - 2\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{2}\right)^{2(n-1)} \\
&= 2\left(\frac{1}{2}\right)^{2(n-1)} - \frac{1}{2} \times 2 \times \left(\frac{1}{2}\right)^{n-1}\left\{1 - \left(\frac{1}{2}\right)^{n-1}\right\} + 1 - 2\left(\frac{1}{2}\right)^{n-1}
\end{aligned}$$

ここで $t = \left(\frac{1}{2}\right)^{n-1}$ とおくと

$$\begin{aligned}
|\overrightarrow{V_n}| &= 2t^2 - t(1 - t) + 1 - 2t \\
&= 2t^2 - t + t^2 + 1 - 2t \\
&= 3t^2 - 3t + 1 \\
&= \boxed{3\left(\frac{1}{2}\right)^{2(n-1)} - 3\left(\frac{1}{2}\right)^{n-1} + 1}
\end{aligned}$$

(4) $\displaystyle\sum_{n=1}^{\infty} |\vec{v_n} - \vec{c}|^2$

$$= \sum_{n=1}^{\infty} \{ |\vec{v_n}|^2 - 2\vec{v_n}\cdot\vec{c} + |\vec{c}|^2 \}$$

$$= \sum_{n=1}^{\infty} \left[3\left(\frac{1}{2}\right)^{2(n-1)} - 3\left(\frac{1}{2}\right)^{n-1} + 1 - 2\times\left\{ 1 - 3\left(\frac{1}{2}\right)^{n} \right\} + 1 \right]$$

$$= \sum_{n=1}^{\infty} \left\{ 3\times\left(\frac{1}{4}\right)^{n-1} - 3\left(\frac{1}{2}\right)^{n-1} + 3\left(\frac{1}{2}\right)^{n-1} + 1 - 2 + 1 \right\}$$

$$= \sum_{n=1}^{\infty} \left\{ 3\times\left(\frac{1}{4}\right)^{n-1} \right\}$$

$$= \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = \frac{3\times4}{3} = \boxed{4}$$