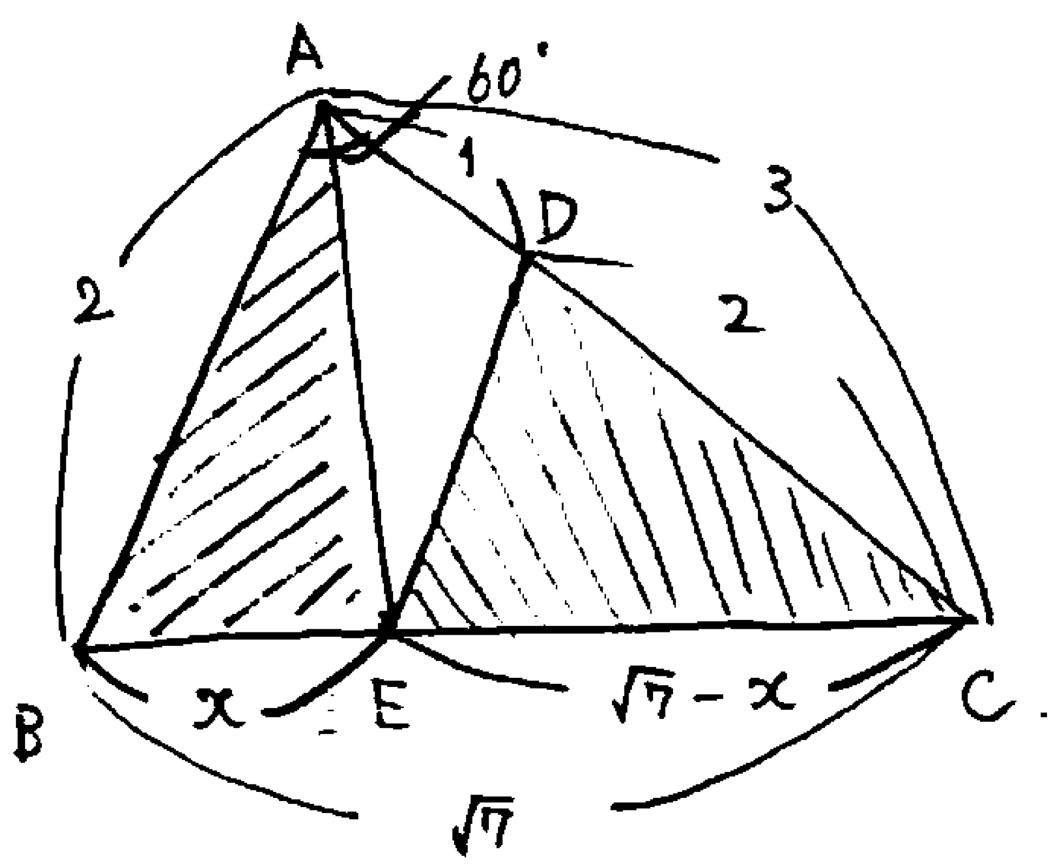

(1) 余弦定理より

$$BC^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos 60^\circ$$
$$= 4 + 9 - 2 \times 2 \times 3 \times \frac{1}{2}$$
$$= 4 + 9 - 6$$
$$= 7$$

よって $\boxed{BC = \sqrt{7}}$

(2) $BE = x$ とおくと $EC = 7 - x$ であり

$\triangle ABC : \triangle DEC = 2 : 1 = 3 \times \sqrt{7} : 2 \times (\sqrt{7} - x)$ より

$$4(\sqrt{7} - x) = 3\sqrt{7}$$
$$4\sqrt{7} - 4x = 3\sqrt{7}$$
$$-4x = -\sqrt{7}$$
$$x = \frac{\sqrt{7}}{4}$$

よって $BE = \frac{\sqrt{7}}{4}$

(3) $\triangle ABE \times \triangle DEC$

$$= (\triangle ABC - \triangle AEC) \times \triangle DEC$$
$$= \left(\triangle ABC - \frac{\sqrt{7} - x}{\sqrt{7}} \triangle ABC\right) \times \frac{2(\sqrt{7} - x)}{3\sqrt{7}} \triangle ABC$$
$$= \frac{\sqrt{7} - \sqrt{7} + x}{\sqrt{7}} \times \triangle ABC \times \frac{2(\sqrt{7} - x)}{3\sqrt{7}} \triangle ABC$$
$$= \frac{2x(\sqrt{7} - x)}{3 \times 7} (\triangle ABC)^2$$
$$= \frac{2}{21} \times (\triangle ABC)^2 \times (-x^2 + \sqrt{7}x)$$
$$= \frac{2}{21} \times \left\{\frac{1}{2} \times 3 \times 2 \times \sin 60^\circ\right\}^2 \times \left\{-\left(x - \frac{\sqrt{7}}{2}\right)^2 + \frac{7}{4}\right\}$$
$$= \frac{2}{21} \times \left(3 \times \frac{\sqrt{3}}{2}\right)^2 \times \left\{-\left(x - \frac{\sqrt{7}}{2}\right)^2 + \frac{7}{4}\right\}$$
$$= \frac{\cancel{2}^{1}}{\cancel{21}_{7}} \times \frac{\cancel{27}^{9}}{\cancel{4}_{2}} \times \left\{-\left(x - \frac{\sqrt{7}}{2}\right)^2 + \frac{7}{4}\right\}$$
$$= \frac{9}{14} \left\{-\left(x - \frac{\sqrt{7}}{2}\right)^2 + \frac{7}{4}\right\}$$

よって $x = \frac{\sqrt{7}}{2}$ で $\triangle ABE \times \triangle DEC$ は最大となりその値は $\frac{9}{14} \times \frac{7}{4} = \frac{9}{8}$ より

$\boxed{BE = \frac{\sqrt{7}}{2}\ \text{のとき}\ T\ \text{は最大値}\ \frac{9}{8}\ \text{をとる}}$