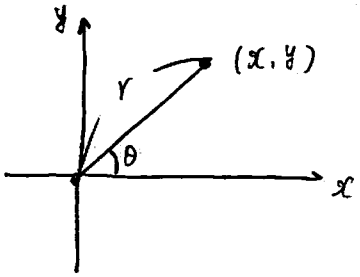
$r = 1-\sin\theta$ より



$$\left.\begin{aligned} x &= (1-\sin\theta)\cos\theta \\ y &= (1-\sin\theta)\sin\theta \end{aligned}\right\} \text{ から}$$

$$\begin{aligned}\frac{dx}{d\theta} &= -\cos^2\theta + (1-\sin\theta)\cdot(-\sin\theta) \\ &= -\cos^2\theta + \sin^2\theta - \sin\theta \\ &= -(1-\sin^2\theta) + \sin^2\theta - \sin\theta \\ &= 2\sin^2\theta - \sin\theta - 1 = (2\sin\theta+1)(\sin\theta-1)\end{aligned}$$

また $\dfrac{dy}{d\theta} = -\cos\theta\sin\theta + (1-\sin\theta)\cos\theta$

$$\begin{aligned}&= \cos\theta - 2\sin\theta\cos\theta \\ &= \cos\theta(1-2\sin\theta)\end{aligned}$$

よって $u(\theta) = \dfrac{dy}{dx} = \boxed{\dfrac{\cos\theta(1-2\sin\theta)}{(2\sin\theta+1)(\sin\theta-1)}}$

$$\lim_{\theta\to+0} u(\theta) = \lim_{\theta\to+0} \frac{\cos\theta(1-2\sin\theta)}{(2\sin\theta+1)(\sin\theta-1)} = \frac{1\times1}{1\times(-1)} = \boxed{-1}$$

$$\lim_{\theta\to\frac{\pi}{2}-0} u(\theta) = \lim_{\theta\to\frac{\pi}{2}-0} \frac{\cos\theta(1-2\sin\theta)}{(2\sin\theta+1)(\sin\theta-1)}$$

$$= \lim_{\theta\to\frac{\pi}{2}-0} \frac{\cos\theta(1-2\sin\theta)(1+\sin\theta)}{(2\sin\theta+1)\times\{-(1-\sin\theta)\}(1+\sin\theta)}$$

$$= \lim_{\theta\to\frac{\pi}{2}-0} \frac{(1-2\sin\theta)(1+\sin\theta)}{-(2\sin\theta+1)\cos\theta} \quad \cdots (*)$$

$$= \lim_{\theta\to\frac{\pi}{2}-0} \left(\frac{-2\sin^2\theta-\sin\theta+1}{-2\sin\theta-1}\times\frac{1}{\cos\theta}\right)$$

ここで $\theta\to\frac{\pi}{2}-0$ のとき $\dfrac{-2\sin^2\theta-\sin\theta+1}{-2\sin\theta-1} \to \dfrac{-2-1+1}{-2-1} = \dfrac{-2}{-3} = \dfrac{2}{3}$ より

$$\lim_{\theta\to\frac{\pi}{2}-0} u(\theta) = \boxed{\infty}$$

(3) (2)の(*)より $u(\theta) = 0$ を $0<\theta<\frac{\pi}{2}$ で解くと $\theta = \dfrac{\pi}{6}$.

よって 増減表は 下のようになる

θ	0	…	$\frac{\pi}{6}$	…	$\frac{\pi}{2}$
$\frac{dy}{dx}$	-1	$-$	0	$+$	∞
(x, y)	$(1, 0)$	↖	$(\frac{\sqrt{3}}{4}, \frac{1}{4})$	↙	$(0, 0)$

$x=\frac{\pi}{6}$ のとき $x = (1-\sin\frac{\pi}{6})\cos\frac{\pi}{6} = (1-\frac{1}{2})\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$y = (1-\sin\frac{\pi}{6})\sin\frac{\pi}{6} = (1-\frac{1}{2})\times\frac{1}{2} = \frac{1}{4}$

ゆえに [y座標が最大である点は $\theta=\frac{\pi}{6}$ のときで $(\frac{\sqrt{3}}{4}, \frac{1}{4})$] であり

グラフの概形は 下のようになる