(1) $\begin{cases} x = \dfrac{\cos t}{1-\sin t} \\ y = \dfrac{\sin t}{1-\cos t} \end{cases}$ $\left(0<t<\dfrac{\pi}{2}\right)$ それぞれをもで微分すると

$$\frac{dx}{dt} = \frac{-\sin t(1-\sin t) - \cos t\cdot(-\cos t)}{(1-\sin t)^2}$$

$$= \frac{-\sin t + \sin^2 t + \cos^2 t}{(1-\sin t)^2}$$

$$= \frac{1-\sin t}{(1-\sin t)^2} = \boxed{\frac{1}{1-\sin t}}$$

$$\frac{dy}{dt} = \frac{\cos t(1-\cos t) - \sin t\cdot\sin t}{(1-\cos t)^2}$$

$$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2}$$

$$= \frac{-(1-\cos t)}{(1-\cos t)^2} = \boxed{-\frac{1}{1-\cos t}}$$

よって $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{-\frac{1}{1-\cos t}}{\frac{1}{1-\sin t}} = \boxed{-\dfrac{1-\sin t}{1-\cos t}}$ より

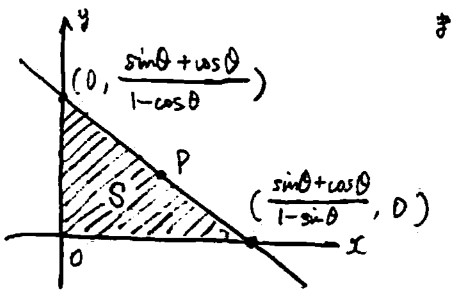
P $\left(\dfrac{\cos\theta}{1-\sin\theta}, \dfrac{\sin\theta}{1-\cos\theta}\right)$ における C の接線 ℓ は

$$y = -\frac{1-\sin\theta}{1-\cos\theta}\left(x - \frac{\cos\theta}{1-\sin\theta}\right) + \frac{\sin\theta}{1-\cos\theta}$$ だから

$$y = -\frac{1-\sin\theta}{1-\cos\theta}x + \frac{\cos\theta\cancel{(1-\sin\theta)}}{(1-\cos\theta)\cancel{(1-\sin\theta)}} + \frac{\sin\theta}{1-\cos\theta}$$

$$\boxed{y = -\frac{1-\sin\theta}{1-\cos\theta}x + \frac{\sin\theta+\cos\theta}{1-\cos\theta}} \quad \text{—①}$$

(2) ① に $x=0$ 代入すると $y = \dfrac{\sin\theta+\cos\theta}{1-\cos\theta}$



また $y=0$ 代入すると $x = \dfrac{\sin\theta+\cos\theta}{1-\sin\theta}$ より

$\left(\dfrac{\alpha}{1-\sin\theta}, 0\right)$, $\left(0, \dfrac{\alpha}{1-\cos\theta}\right)$ を通るので

求める面積 S は

$$S = \frac{1}{2}\times\frac{\alpha}{1-\sin\theta}\times\frac{\alpha}{1-\cos\theta}$$

$$= \frac{1}{2}\times\frac{\alpha^2}{1-\alpha+\sin\theta\cos\theta}$$

$$= \frac{\alpha^2}{2-2\alpha+2\sin\theta\cos\theta}$$

ここで $\sin\theta+\cos\theta = \alpha$ より

$(\sin\theta+\cos\theta)^2 = \alpha^2$ から

$1+2\sin\theta\cos\theta = \alpha^2$

$2\sin\theta\cos\theta = \alpha^2-1$.

よって $S = \dfrac{\alpha^2}{2-2\alpha+\alpha^2-1}$

$$= \frac{\alpha^2}{\alpha^2-2\alpha+1}$$

$$= \boxed{\left(\frac{\alpha}{\alpha-1}\right)^2}$$

(3) $S = \left(\dfrac{\alpha}{\alpha-1}\right)^2$

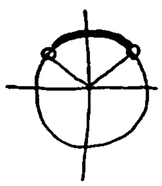
$= \left(\dfrac{\alpha-1+1}{\alpha-1}\right)^2$

$= \left(1+\dfrac{1}{\alpha-1}\right)^2$ であり．

$\alpha = \sin\theta + \cos\theta$

$= \sqrt{2}\sin\left(\theta+\dfrac{\pi}{4}\right)$ だから　$0<\theta<\dfrac{\pi}{2}$ のとき

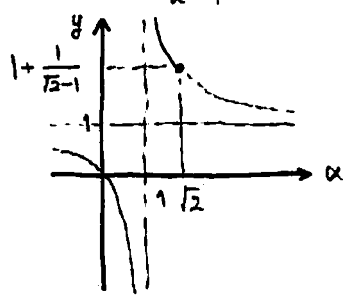
$\dfrac{\pi}{4} < \theta+\dfrac{\pi}{4} < \dfrac{3}{4}$ より



$\dfrac{1}{\sqrt{2}} < \sin\left(\theta+\dfrac{\pi}{4}\right) \leqq 1$

よって　$1 < \alpha \leqq \sqrt{2}$　であるから

$y = 1+\dfrac{1}{\alpha-1}$　$(1<\alpha\leqq\sqrt{2})$ のグラフは 下のようになるので



$1+\dfrac{1}{\alpha-1} \geqq 1+\dfrac{1}{\sqrt{2}-1} = \dfrac{\sqrt{2}}{\sqrt{2}-1} = \sqrt{2}(\sqrt{2}+1) = 2+\sqrt{2}$

よって　$S \geqq (2+\sqrt{2})^2$ より

$\boxed{S \geqq 6+4\sqrt{2}}$