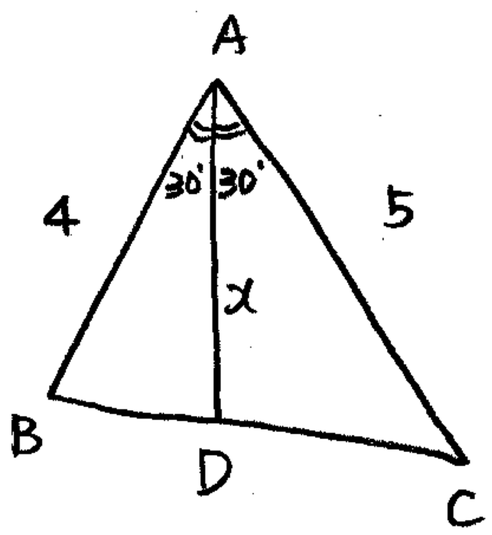

(1) 余弦定理より

$$BC^2 = 4^2 + 5^2 - 2\times 4\times 5\times \cos 60^\circ$$
$$= 16 + 25 - 40\times\frac{1}{2}$$
$$= 41 - 20$$
$$= 21$$

よって $\boxed{BC = \sqrt{21}}$

(2) $AD = x$ とおくと $\triangle ABC = \triangle ABD + \triangle ACD$ より

$$\frac{1}{2}\times 4\times 5\times \sin 60^\circ = \frac{1}{2}\times 4\times x\times \sin 30^\circ + \frac{1}{2}\times 5\times x\times \sin 30^\circ$$ だから

$$\frac{1}{2}\times 20\times \frac{\sqrt{3}}{2} = \frac{1}{2}\times 4x\times \frac{1}{2} + \frac{1}{2}\times 5x\times \frac{1}{2}$$

よって $20\sqrt{3} = 4x + 5x$ より $AD = x = \boxed{\frac{20\sqrt{3}}{9}}$

(3) $BD : DC = AB : AC = 4 : 5$ より

$$BD = \frac{4}{9}BC = \boxed{\frac{4\sqrt{21}}{9}}$$