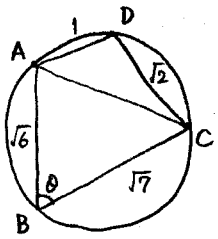

余弦定理より

$$\begin{cases} AC^2 = 6+7-2\times\sqrt{6}\times\sqrt{7}\times\cos B \\ AC^2 = 2+1-2\times\sqrt{2}\times 1\times\cos D \\ \cos B = -\cos D \end{cases}$$ がなりたつ。

よって $$\begin{cases} AC^2 = 13-2\sqrt{42}\cos B \\ AC^2 = 3+2\sqrt{2}\cos B \end{cases}$$ より

$13-2\sqrt{42}\cos B = 3+2\sqrt{2}\cos B$ だから

$$2\sqrt{2}(1+\sqrt{21})\cos B = 10$$

$$\cos B = \frac{10}{2\sqrt{2}(1+\sqrt{21})} = \frac{5\sqrt{2}(1-\sqrt{21})}{2(1-21)} = \frac{5\sqrt{2}(1-\sqrt{21})}{-2\times 20} = \frac{\sqrt{2}(\sqrt{21}-1)}{8}$$

よって $$AC^2 = 3+2\sqrt{2}\times\frac{\sqrt{2}(\sqrt{21}-1)}{8} = 3+\frac{\sqrt{21}-1}{2} = \frac{①\ 5+\sqrt{21}\ ②}{2}$$ だから

$$AC = \sqrt{\frac{5+\sqrt{21}}{2}} = \sqrt{\frac{10+2\sqrt{21}}{4}} = \frac{\sqrt{3}+\boxed{\sqrt{7}}^{③}}{2}$$ となる

また $\sin^2\theta = \sin^2 B$

$$= 1-\cos^2 B = 1-\left\{\frac{\sqrt{2}(\sqrt{21}-1)}{8}\right\}^2 = 1-\frac{2(21-2\sqrt{21}+1)}{64} = 1-\frac{22-2\sqrt{21}}{32} = 1-\frac{11-\sqrt{21}}{16} = \boxed{\frac{5+\sqrt{21}}{16}}$$ ④ だから

$$\sin^2\theta = \frac{1}{8}\times\frac{5+\sqrt{21}}{2} = \frac{AC^2}{\boxed{8}}$$ ⑤ となり

円の半径を R とすると $2R = \frac{AC}{\sin\theta}$ より

$$4R^2 = \frac{AC^2}{\sin^2\theta} = \frac{AC^2}{\frac{AC^2}{8}} = 8$$

よって $R^2 = 2$ より 円の半径 R は $\boxed{\sqrt{2}}$ ⑥ となる

また四角形の面積は

$$\sin\theta = \sqrt{\frac{AC^2}{8}} = \frac{AC}{2\sqrt{2}} = \frac{\sqrt{3}+\sqrt{7}}{4\sqrt{2}} = \frac{\sqrt{6}+\sqrt{14}}{8}$$

をつかうと

$\triangle ABC + \triangle DAC$

$$= \frac{1}{2}\times\sqrt{6}\times\sqrt{7}\sin\theta + \frac{1}{2}\times 1\times\sqrt{2}\times\sin\theta$$

$$= \frac{\sqrt{2}}{2}\times(\sqrt{21}+1)\sin\theta$$

$$= \frac{\sqrt{2}}{2}\times(\sqrt{21}+1)\times\frac{\sqrt{2}(\sqrt{3}+\sqrt{7})}{8}$$

$$= \frac{1}{8}(\sqrt{21}+1)(\sqrt{3}+\sqrt{7})$$

$$= \frac{1}{8}(\sqrt{63}+\sqrt{3}+\sqrt{21\times 7}+\sqrt{7})$$

$$= \frac{1}{8}\times(3\sqrt{7}+\sqrt{3}+7\sqrt{3}+\sqrt{7})$$

$$= \frac{1}{8}(8\sqrt{3}+4\sqrt{7})$$

$$= \boxed{\frac{2\sqrt{3}+\sqrt{7}}{2}}^{⑦}$$ となる