(1) $x = r\cos\theta = \sqrt{\cos 2\theta}\cos\theta$

$y = r\sin\theta = \sqrt{\cos 2\theta}\sin\theta$ 　よって Ｃ上の点 $P(x, y)$ は

$$\boxed{\left(\sqrt{\cos 2\theta}\cos\theta,\ \sqrt{\cos 2\theta}\sin\theta\right)}$$

(2) $$\frac{dx}{d\theta} = \frac{-2\sin 2\theta}{2\sqrt{\cos 2\theta}} \times \cos\theta + \sqrt{\cos 2\theta} \times (-\sin\theta)$$

$$= \frac{-\sin 2\theta\cos\theta - \cos 2\theta\sin\theta}{\sqrt{\cos 2\theta}} = -\frac{\sin 3\theta}{\sqrt{\cos 2\theta}}$$

$$\frac{dy}{d\theta} = \frac{-2\sin 2\theta}{2\sqrt{\cos 2\theta}} \times \sin\theta + \sqrt{\cos 2\theta}\cos\theta$$

$$= \frac{-\sin 2\theta\sin\theta + \cos 2\theta\cos\theta}{\sqrt{\cos 2\theta}} = \frac{\cos 3\theta}{\sqrt{\cos 2\theta}}$$

よって $$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{\cos 3\theta}{\sqrt{\cos 2\theta}}}{\frac{-\sin 3\theta}{\sqrt{\cos 2\theta}}} = -\frac{\cos 3\theta}{\sin 3\theta} = -\frac{1}{\tan 3\theta}$$ であり

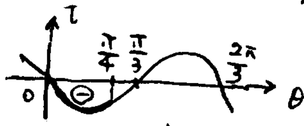
この値が -1 のとき，$-\frac{1}{\tan 3\theta} = -1$ だから

$\tan 3\theta = 1$ である

ここで $0 \leqq 3\theta \leqq \frac{3}{4}\pi$ でとけばよいから $3\theta = \frac{\pi}{4}$ より

$\boxed{\theta = \frac{\pi}{12}}$ となる

(3) $\frac{dx}{d\theta} = -\frac{\sin 3\theta}{\sqrt{\cos 2\theta}}$ は

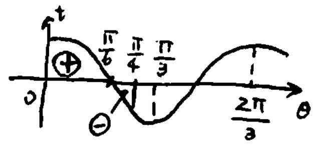


$t = -\sin 3\theta$

$0 < \theta < \frac{\pi}{4}$ で必ず負となる

よって x は単調減少

$\frac{dy}{d\theta} = \frac{\cos 3\theta}{\sqrt{\cos 2\theta}}$ は



$t = \cos 3\theta$

$0 < \theta < \frac{\pi}{6}$ で 正

$\theta = \frac{\pi}{6}$ で 0

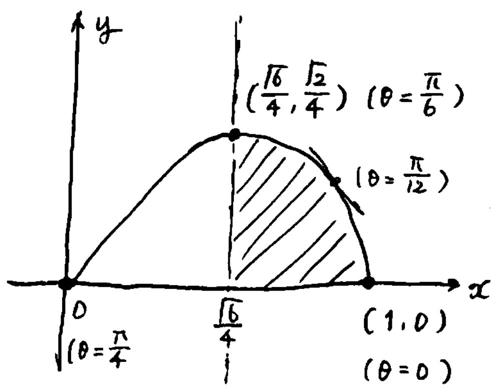
$\frac{\pi}{6} < \theta < \frac{\pi}{4}$ で負

θ	0	$\cdots$	$\frac{\pi}{6}$	$\cdots$	$\frac{\pi}{4}$
$\frac{dx}{d\theta}$		$-$	$-$	$-$	
$\frac{dy}{d\theta}$		$+$	0	$-$	
(x, y) とその向き	$(1, 0)$	↖	$\left(\frac{\sqrt{6}}{4}, \frac{\sqrt{2}}{4}\right)$	↙	$(0, 0)$

$\theta = \frac{\pi}{6}$ で $x = \sqrt{\cos\frac{\pi}{3}} \cdot \cos\frac{\pi}{6} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{4}$

$y = \sqrt{\cos\frac{\pi}{3}} \cdot \sin\frac{\pi}{6} = \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{2}}{4}$

よって Ｃの概形は以下のようになる



ゆえに求める面積は斜線部の面積であり、これをSとおくと

$$S=\int_{\frac{\sqrt{6}}{4}}^{1} y\,dx=\int_{\frac{\pi}{6}}^{0}\sqrt{\cos 2\theta}\sin\theta\cdot\left(-\frac{\sin 3\theta}{\sqrt{\cos 2\theta}}\right)d\theta$$

$$=\int_{\frac{\pi}{6}}^{0}(-\sin 3\theta\sin\theta)\,d\theta$$

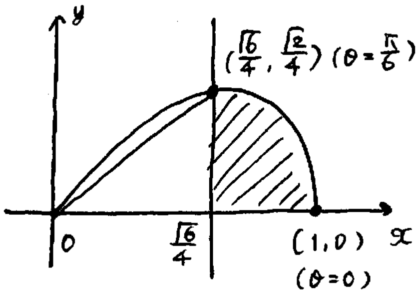
$$=\frac{1}{2}\int_{\frac{\pi}{6}}^{0}(\cos 4\theta-\cos 2\theta)\,d\theta$$

$$=\frac{1}{2}\left[\frac{1}{4}\sin 4\theta-\frac{1}{2}\sin 2\theta\right]_{\frac{\pi}{6}}^{0}$$

$$=-\frac{1}{8}\sin\frac{2}{3}\pi+\frac{1}{4}\sin\frac{\pi}{3}$$

$$=-\frac{1}{8}\times\frac{\sqrt{3}}{2}+\frac{1}{4}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}\times\frac{1}{8}=\boxed{\frac{\sqrt{3}}{16}}$$ となる

別解



$$S=\int_{0}^{\frac{\pi}{6}}\frac{1}{2}r^2\,d\theta-\frac{1}{2}\times\frac{\sqrt{6}}{4}\times\frac{\sqrt{2}}{4}$$

$$=\frac{1}{2}\int_{0}^{\frac{\pi}{6}}\cos 2\theta\,d\theta-\frac{2\sqrt{3}}{32}$$

$$=\left[\frac{1}{4}\sin 2\theta\right]_{0}^{\frac{\pi}{6}}-\frac{\sqrt{3}}{16}$$

$$=\frac{1}{4}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{16}$$

$$=\boxed{\frac{\sqrt{3}}{16}}$$ となる