

$$s = \alpha + \beta + r, \quad t = \alpha\beta + \beta r + r\alpha, \quad u = \alpha\beta r \quad \text{より}$$

$$\begin{aligned} (1) & (\alpha + \beta)(\beta + r)(r + \alpha) \\ &= (s - r)(s - \alpha)(s - \beta) \\ &= s^3 - (\alpha + \beta + r)s^2 + (\alpha\beta + \beta r + r\alpha)s - \alpha\beta r \\ &= s^3 - s \times s^2 + ts - u \\ &= \boxed{ts - u} \end{aligned}$$

$$(2) \quad |\alpha| = |\beta| = |r| = 1 \quad \text{より} \quad |\alpha|^2 = |\beta|^2 = |r|^2 = 1 \quad \text{だから}$$

$$\alpha \bar{\alpha} = \beta \bar{\beta} = r \bar{r} = 1$$

また α, β, r はいずれも複素数平面上での
原点までの距離が1より

$\alpha \neq 0$ かつ $\beta \neq 0$ かつ $r \neq 0$ であるから

$$\bar{\alpha} = \frac{1}{\alpha}, \quad \bar{\beta} = \frac{1}{\beta}, \quad \bar{r} = \frac{1}{r} \quad \text{がなりたつ}$$

$$\begin{aligned} (i) \quad \text{よって} \quad \bar{s} &= \overline{\alpha + \beta + r} = \bar{\alpha} + \bar{\beta} + \bar{r} \\ &= \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{r} \\ &= \frac{\beta r + r\alpha + \alpha\beta}{\alpha\beta r} = \frac{t}{u} \quad \text{より} \quad \boxed{\bar{s} = \frac{t}{u}} \quad \text{がなりたつ} \quad -① \end{aligned}$$

$$\begin{aligned} \text{また} \quad \bar{t} &= \overline{\alpha\beta + \beta r + r\alpha} = \bar{\alpha}\bar{\beta} + \bar{\beta}\bar{r} + \bar{r}\bar{\alpha} \\ &= \frac{1}{\alpha} \times \frac{1}{\beta} + \frac{1}{\beta} \times \frac{1}{r} + \frac{1}{r} \times \frac{1}{\alpha} \\ &= \frac{r + \alpha + \beta}{\alpha\beta r} = \frac{s}{u} \quad \text{より} \quad \boxed{\bar{t} = \frac{s}{u}} \quad \text{がなりたつ} \quad -② \end{aligned}$$

$$\begin{aligned} \text{また} \quad \bar{u} &= \overline{\alpha\beta r} = \bar{\alpha} \times \bar{\beta} \times \bar{r} \\ &= \frac{1}{\alpha} \times \frac{1}{\beta} \times \frac{1}{r} = \frac{1}{u} \quad \text{より} \quad \boxed{\bar{u} = \frac{1}{u}} \quad \text{がなりたつ} \quad -③ \end{aligned}$$

$$(ii) \quad |s|^2 = s\bar{s} = s \times \frac{t}{u} \quad (① \text{より})$$

$$\text{また} \quad |t|^2 = t\bar{t} = t \times \frac{s}{u} \quad (② \text{より}) \quad \text{よって} \quad |s|^2 = |t|^2 \quad \text{より} \quad \boxed{|s| = |t|} \quad \text{がなりたつ}$$

$$(iii) \quad \frac{(\alpha + \beta)(\beta + r)(r + \alpha)}{u} = \frac{ts - u}{u} \quad (① \text{より})$$

$$= s \cdot \frac{t}{u} - 1$$

$$= s \cdot \bar{s} - 1 \quad ((2)(i) \text{の} ① \text{より})$$

$$= |s|^2 - 1 \quad \text{よって} \quad |s|^2 - 1 \quad \text{は実数となるので}$$

$$\frac{(\alpha + \beta)(\beta + r)(r + \alpha)}{u} \quad \text{は実数である}$$